Impacts from Initialization techniques – An optimal computational resource allocation problem

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Background & Purpose

- Initialization techniques are always considered as "computational-resource-free"
- Not true under computational expensive environment where single FE costs a lot
- Optimally allocate the limited computational resources becomes important

Optimal Computational Resource Allocation Problem (OCRAP):

Under given amount of computational resources (*R*), objective function of the base problem (f), initialization technique (*IniT*) and optimization algorithm (*OA*), to find an resource allocation scheme (RA=IniR/(IniR+OptR)) where *IniR* and *OptR* are the resources consumed by *IniT* and *OA* so that either the optimal solution (y=y*) of the objective function (y=f(x)) is achieved with the least total resources (IniR+OptR<R) or the best suboptimal solution (y!=y*) is achieved when resources are used up (IniR+OptR=R).

Computational resource is defined as number of FE used under computational expensive environment. Due to the extreme long time required by FE, other calculations are negligible.

Problem Formulation:

- General version:

$$\min_{IniT,OA,f(\bullet)} \{(IniR + OptR), |y - y^*|\} = F(RA)$$
$$s.t.\ IniR + OptR \le R$$
$$IniR > 0$$
$$OptR > 0$$

- Simulation version:

$$\min_{IniT,OA,f(\bullet)} \{(IniFE + OptFE), |y - y^*|\} = F(RA)$$
$$s.t.\ IniFE + OptFE \le TotalFE$$
$$IniFE > 0$$
$$OptFE > 0$$

Simulation cases

Initialization techniques:

- Pseudo Random Number Generator (PRNG)
- Opposition-Based Learning (OBL) [1]
- Quasi-Opposition-Based Learning (QOBL) [2]
- Quadratic Interpolation (QI) [3]

Optimization algorithms:

- Differential Evolution (DE) [4]
- Chemical Reaction Optimization (CRO) [5]

Benchmark functions:

- CEC14 computational expensive problem set

CEC14 COMPUTATIONAL EXPENSIVE PROBLEM SET

Functions	Search Ranges	
Shifted Sphere function	$[-20, 20]$	$f_1(x) = \sum_{i=1}^{D} {y_i}^2,\ y = x - o_1$
Shifted Ellipsoid function	$[-20, 20]$	$f_2(x) = \sum_{i=1}^{D} i{y_i}^2,\ y = x - o_2$
Shifted Rotated Ellipsoid function	$[-20, 20]$	$f_3(x) = \sum_{i=1}^{D} i{y_i}^2,\ y = M_3(x - o_3)$
Shifted Step function	$[-20, 20]$	$f_4(x) = \sum_{i=1}^{D} (\lfloor y_i + 0.5 \rfloor)^2,\ y = x - o_4$
Shifted Ackley's function	$[-32, 32]$	$f_5(x) = -20\exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} {y_i}^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi y_i)\right) + 20 + e,\ y = x - o_5$
Shifted Griewank's function	$[-600, 600]$	$f_6(x) = \sum_{i=1}^{D} \frac{(y_i)^2}{4000} - \prod_{i=1}^{D}\cos\left(\frac{y_i}{\sqrt{i}}\right) + 1,\ y = x - o_6$
Shifted Rotated Rosenbrock's function	$[-20, 20]$	$f_7(x) = \sum_{i=1}^{D-1} \left(100(y_i^2 - y_{i+1})^2 + (y_i - 1)^2\right), y = M_7\left(\frac{2.048(x-o_7)}{20}\right) + 1$
Shifted Rotated Rastrigin's function	$[-20, 20]$	$f_8(x) = \sum_{i=1}^{D} \left(y_i^2 - 10\cos(2\pi y_i) + 10\right),\ y = M_8\frac{5.12(x-o_8)}{20}$

Metrics:

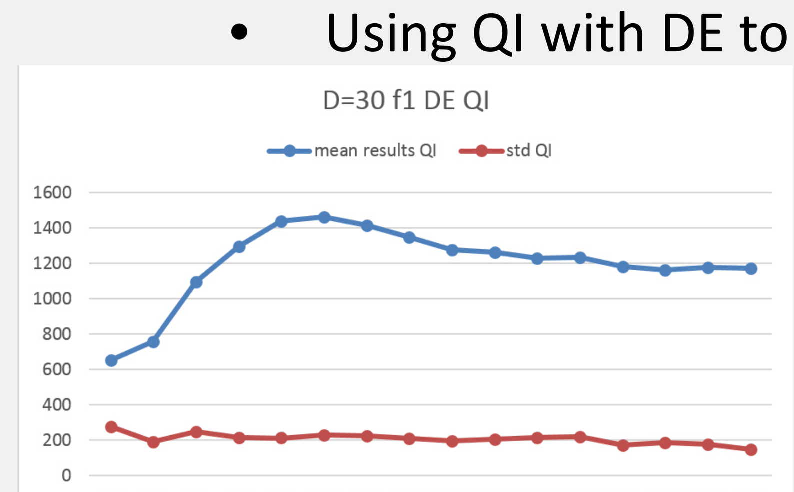
- Without *IniR* considered (traditional way)
- Considering *IniR*
- Solve the OCRAP

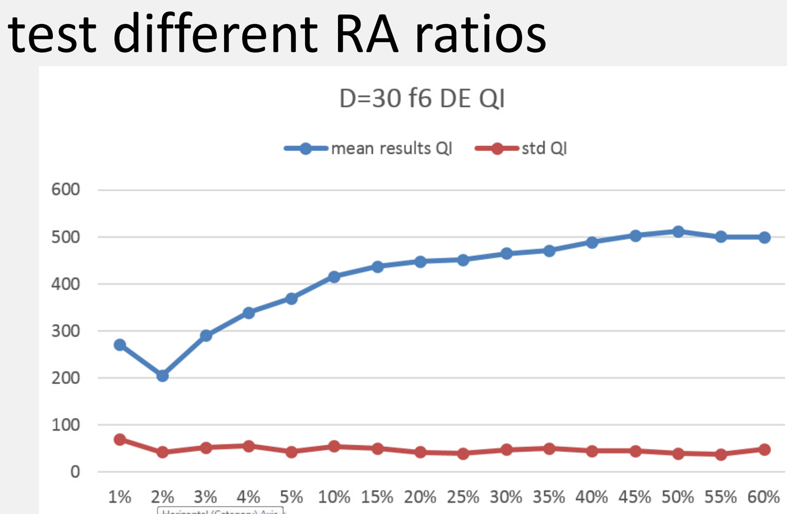
Simulation results

- Notations & settings:
  - M1: without considering *IniR*
  - M2: *IniR* considered
  - M3 = r_init/r_rand, the ratio between results from using *IniT* and using PRNG.
    M3<1 means *IniT* better
    M3>1 means PRNG better
  - D=10,30 & 100; MaxFE(R)=50*D; No. of run=50
- Comparison between *IniR* considered and not considered:

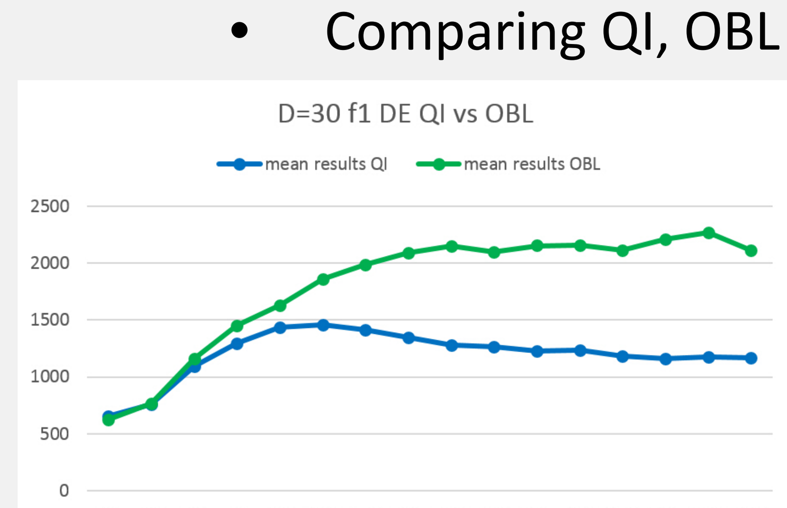
Dimension	Function	*OBL*				*QOBL*				*QI*			
		M1		*M2*		*M1*		*M2*		*M1*		*M2*	
		mean	*M3*	mean	*M3*	mean	*M3*	mean	*M3*	mean	*M3*	mean	*M3*
$D = 10$	f_1	292.688	**0.896**	320.001	0.98	303.638	**0.93**	341.753	1.047	206.56	**0.633**	225.633	0.694
	f_2	1425.08	**0.867**	1678.31	1.022	1651.54	**1.005**	1660.1	1.01	814.83	0.496	811.026	**0.494**
	f_3	1829.25	**0.929**	2034.93	1.033	2032.96	**1.032**	2230.04	1.133	1038.26	**0.527**	1120.04	0.569
	f_4	305.76	**0.979**	330.92	1.06	311.94	**0.999**	329.26	1.055	182.32	**0.584**	198.8	0.637
	f_5	18.624	**1.002**	19.8205	1.067	18.567	**0.999**	18.807	1.012	16.499	0.888	16.3709	**0.881**
	f_6	79.4	**0.926**	82.361	0.961	87.439	**1.02**	92.705	1.082	73.167	**0.854**	78.779	0.919
	f_7	798.859	**0.88**	883.785	**0.973**	904.741	0.996	1108.49	1.221	506.106	**0.557**	592.475	0.652
	f_8	102.563	1.013	101.49	**1.003**	99.74	**0.985**	101.496	1.003	85.656	**0.846**	88.84	0.878
$D = 30$	f_1	1952.7	**0.974**	2064.7	1.03	2039.07	**1.017**	2115.44	1.055	1373.82	0.685	1331.04	**0.664**
	f_2	27010.3	**0.946**	27700.6	0.97	27624.2	**0.968**	28722	1.006	17210.8	**0.603**	17850.1	0.625
	f_3	42243	1.002	41434.4	**0.983**	41594.8	**0.986**	44373.7	1.052	26003.1	0.617	25415.6	**0.603**
	f_4	2000.08	**1.003**	2007.02	1.007	2014.52	**1.01**	2037.34	1.022	1156.22	**0.58**	1210.7	0.607
	f_5	20.15	**0.998**	20.1741	1	20.1708	**0.999**	20.2353	1.003	18.734	0.928	18.4395	**0.914**
	f_6	444.645	**0.972**	455.463	0.995	463.495	**1.013**	481.687	1.053	441.425	**0.965**	455.731	0.996
	f_7	12117.4	**0.933**	12914.4	0.995	13199.3	**1.017**	14311.4	1.102	7357.16	**0.567**	8023.41	0.618
	f_8	454.58	**0.977**	460.919	0.991	459.388	**0.987**	467.537	1.005	396.179	0.851	392.518	**0.844**
$D = 100$	f_1	9951.52	1.005	9935.93	**1.004**	9953.54	**1.006**	10128.4	1.023	6433.26	**0.65**	6453.96	0.652
	f_2	462924	**1.003**	469329	1.017	464419	**1.006**	467461	1.013	294942	0.639	290347	**0.629**
	f_3	704188	**0.993**	713346	1.006	702866	**0.992**	721093	1.017	472779	**0.667**	481583	0.679
	f_4	9741.92	**0.994**	9906.98	1.011	9819.86	**1.002**	10106.1	1.031	6172.86	**0.63**	6208.14	0.633
	f_5	20.744	1	20.754	1	20.751	1	20.7547	1	19.814	0.955	19.8229	0.955
	f_6	1937.39	**0.937**	1952.19	0.944	2072.97	**1.002**	2110.65	1.021	2047.43	**0.99**	2083.3	1.007
	f_7	114970	**0.995**	119493	1.034	117515	**1.017**	120207	1.04	66601.9	**0.576**	67878.6	0.587
	f_8	1936.96	**0.998**	1941.86	1.001	1941.51	**1.001**	1961.03	1.011	1631.54	0.841	1611.6	**0.831**

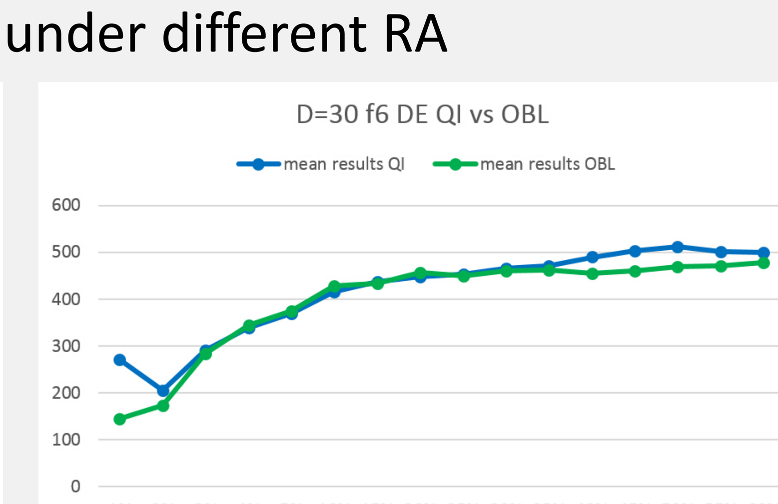
- Some curves
  - Using QI with DE to test different RA ratios

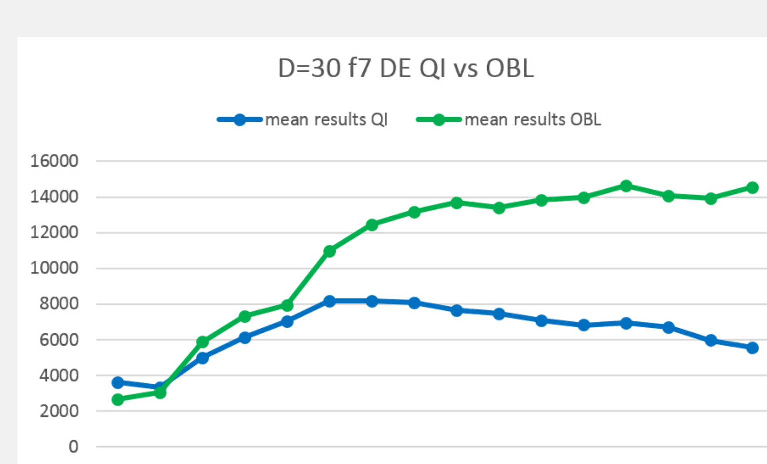


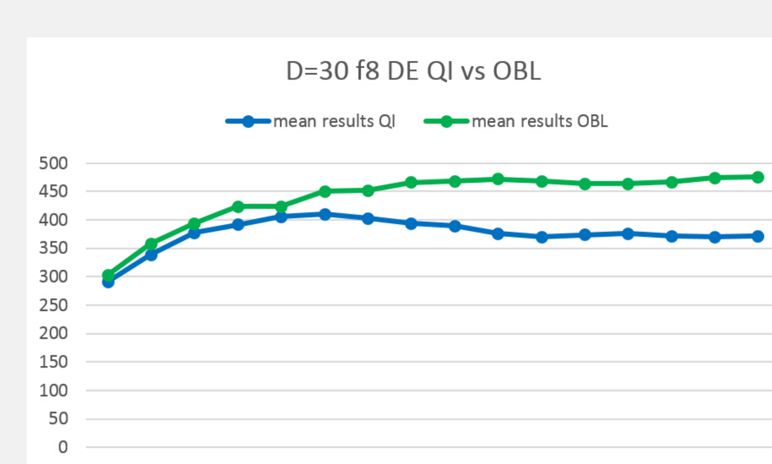


  - Comparing QI, OBL under different RA









Conclusion

- Formulate and solve the optimal computational resource allocation problem
- Define the computational resource under the expensive environment
- Conduct simulations analyze performances from different initialization techniques

Reference

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